

The $a_0K^+K^-$ -vertex in light cone QCD sum rules

A. Gokalp ^{*} and O. Yilmaz [†]

Physics Department, Middle East Technical University, 06531 Ankara, Turkey

(February 1, 2008)

Abstract

We investigate the $a_0K^+K^-$ -vertex in the framework of light cone QCD sum rules. We estimate the coupling constant $g_{a_0K^+K^-}$ which is an essential ingredient in the analysis of physical processes involving $a_0(980)$ meson. Our result is somewhat larger than the previous determinations of this coupling constant.

PACS numbers: 12.38.Lg;13.25.Jx;14.40.Cs

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^{*}agokalp@metu.edu.tr

[†]oyilmaz@metu.edu.tr

The structure of light scalar mesons $f_0(980)$ and $a_0(980)$ has been a controversial problem in hadron spectroscopy. In the naive quark model $q\bar{q}$ [1], $a_0(980)$ can be interpreted as $a_0 = (u\bar{u} - d\bar{d})/\sqrt{2}$. On the other hand, the strong coupling of $f_0(980)$ to kaons suggests the structure $f_0 = s\bar{s}$ for this state. However, then the almost exact degeneracy of the masses of $a_0(980)$ and $f_0(980)$ cannot be explained. In order to explain the properties of these states several proposals have been put forward over the years. A four quark $q^2\bar{q}^2$ state interpretation with symbolic quark structure $f_0 = s\bar{s}(u\bar{u} + d\bar{d})/\sqrt{2}$ and $a_0 = s\bar{s}(u\bar{u} - d\bar{d})/\sqrt{2}$ was proposed in the framework of MIT-bag model where the scalar meson states are spatially compact [2]. Another possibility about the structure of $a_0(980)$ and $f_0(980)$ was suggested where these meson states are considered to be bound states of hadrons. This possibility is referred to as their being $K\bar{K}$ molecules in which case they are considered as spatially extended objects [3]. Furthermore, some analyses suggest the qualitative picture that these scalar meson states have a compact $q^2\bar{q}^2$ structure that spends some part of its lifetime in the $K\bar{K}$ meson system [4].

The radiative decays of $\phi(1020)$ meson $\phi \rightarrow a_0\gamma$ and $\phi \rightarrow f_0\gamma$ provide important tests to distinguish among the different possibilities about the structure of $a_0(980)$ and $f_0(980)$ scalar meson states [4,5]. It is generally agreed that the experimental data supports the kaon loop mechanism for these decays in both the $q^2\bar{q}^2$ state and $K\bar{K}$ molecule models where these radiative decays proceed by photon emission from an intermediate K^+K^- loop [5]. Moreover, the study of the reactions $\phi \rightarrow \pi^0\pi^0\gamma$ and $\phi \rightarrow \pi^0\eta\gamma$ and of the interference patterns in these reactions have been used to develop arguments about the structure of $a_0(980)$ and $f_0(980)$ states [6]. In the analyses involving $a_0(980)$ meson the strong coupling constant $g_{a_0K^+K^-}$ plays an important role.

In this work, we estimate the coupling constant $g_{a_0K^+K^-}$ by employing light cone QCD sum rules. This method has been applied to study hadronic properties and in particular it has been used for the calculation of hadronic coupling constants [7].

In order to study the $a_0K^+K^-$ -vertex and to estimate the coupling constant $g_{a_0K^+K^-}$ we consider the two-point correlation function

$$T_\mu(p, q) = i \int d^4x e^{ip \cdot x} \langle K^+(q) | T \{ j_\mu^K(x) j^{a_0}(0) \} | 0 \rangle \quad (1)$$

with p and j_μ^K the four-momentum and the interpolating current for K^- meson, j^{a_0} the interpolating current for a_0 meson, and q the four-momentum of K^+ state. The interpolating quark currents are the axial vector $j_\mu^K = \bar{u}\gamma_\mu\gamma_5 s$ and the scalar $j^{a_0} = (\bar{u}u - \bar{d}d)/2$ current. The scalar current j^{a_0} is assumed to have a non-vanishing matrix element between the vacuum and $a_0(980)$ meson state, $\langle a_0 | j^{a_0} | 0 \rangle = \lambda_{a_0}$, where λ_{a_0} is called the overlap amplitude which was determined by QCD sum rules method [8], and this particular choice for the current in terms of quark fields does not imply the pure $(\bar{u}u - \bar{d}d)/\sqrt{2}$ structure for $a_0(980)$ meson.

The correlation function can be written in terms of two independent invariant functions T_1 and T_2 as

$$T_\mu(p, q) = iT_1 \left((p+q)^2, p^2 \right) p_\mu + T_2 \left((p+q)^2, p^2 \right) q_\mu \quad (2)$$

We consider the invariant function T_1 . In order to construct the theoretical part of the sum rule for the coupling constant $g_{a_0K^+K^-}$ we calculate the function T_1 in terms of QCD degrees

of freedom by evaluating the correlation function in the deep Euclidean region where p^2 and $(p+q)^2$ are large and negative as an expansion near the light cone $x^2 = 0$. This expansion involves matrix elements of non-local composite operators between kaon and vacuum states which defines kaon distribution amplitudes of increasing twist. We retain terms up to twist four accuracy since higher twist amplitudes are known to make a small contribution [9]. In our calculation we use the full light propagator with both perturbative and nonperturbative contributions which is given as [10]

$$\begin{aligned} iS(x, 0) &= \langle 0 | T \{ \bar{q}(x) q(0) \} | 0 \rangle \\ &= i \frac{\not{x}}{2\pi^2 x^4} - \frac{\langle \bar{q}q \rangle}{12} - \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle \\ &\quad - i g_s \frac{1}{16\pi^2} \int_0^1 du \left\{ \frac{\not{x}}{x^2} \sigma_{\mu\nu} G^{\mu\nu}(ux) - 4iu \frac{x_\mu}{x^2} G^{\mu\nu}(ux) \gamma_\nu \right\} + \dots \end{aligned} \quad (3)$$

After a straightforward computation and performing the Fourier transforms we obtain

$$\begin{aligned} T_1(p^2, (p+q)^2) &= \frac{f_K M_K^2}{2(6m_s)} \int_0^1 du \varphi_{\sigma K}(u) \frac{2[p + (1-u)q] \cdot q}{\{[p + (1-u)q]^2\}^2} \\ &\quad + \frac{1}{2} \frac{f_K M_K^2}{m_s} \int_0^1 du \varphi_{pK}(u) \frac{1}{[p + (1-u)q]^2} \\ &\quad + \frac{f_{3K}}{2} \int_0^1 dv \int D\alpha_i \varphi_{3K}(\alpha_i) \frac{M_K^2}{\{[p + (1-\alpha_1 - v\alpha_3)q]^2\}^2} (2v-1) \end{aligned} \quad (4)$$

In this expression the functions $\varphi_{\sigma K}$ and φ_{pK} are the twist 3 quark-antiquark kaon distribution amplitudes defined by the matrix elements [11]

$$\langle K(q) | \bar{u}(x) i\gamma_5 s(0) | 0 \rangle = f_K \mu_K \int_0^1 du e^{iuq \cdot x} \varphi_{pK}(u) \quad , \quad (5)$$

$$\langle K(q) | \bar{u}(x) \sigma_{\mu\nu} \gamma_5 s(0) | 0 \rangle = i \frac{f_K \mu_K}{6} \left(1 - \frac{M_K^2}{\mu_K^2} \right) (q_\mu x_\nu - x_\mu q_\nu) \int_0^1 du e^{iuq \cdot x} \varphi_{\sigma K}(u) \quad , \quad (6)$$

where $\mu_K = M_K^2/m_s$ is the twist 3 distribution amplitude normalization factor and we put $m_u = m_d = 0$. We work in the gauge $x^\mu A_\mu = 0$, consequently the path-ordered gauge factor is not included in the matrix elements. The twist 3 quark-antiquark-gluon kaon distribution amplitude φ_{3K} is defined as [11]

$$\begin{aligned} \langle K(q) | \bar{u}(x) \sigma_{\alpha\beta} g_s G_{\mu\nu}(vx) s(0) | 0 \rangle &= i f_{3K} [(q_\mu q_\alpha g_{\nu\beta} - q_\nu q_\alpha g_{\mu\beta}) - (q_\mu q_\beta g_{\nu\alpha} - q_\nu q_\beta g_{\mu\alpha})] \\ &\quad \times \int D\alpha_i \varphi_{3K}(\alpha_i) e^{iuq \cdot x(\alpha_1 + v\alpha_3)} \end{aligned} \quad (7)$$

where $D\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$. After performing the Borel transformation with respect to the variables $Q_1^2 = -(p+q)^2$ and $Q_2^2 = -p^2$, we obtain the theoretical expression for the invariant function in the form

$$\begin{aligned} T_1(M_1^2, M_2^2) &= \frac{f_K M_K^2 M^2}{2m_s} [-\varphi_{pK}(u_0) + \frac{1}{6} \varphi'_{\sigma K}(u_0)] \\ &\quad + \frac{f_{3K} M_K^2}{2} \int_0^{u_0} d\alpha_1 \int_{u_0 - \alpha_1}^{1 - \alpha_1} \frac{d\alpha_3}{\alpha_3} \varphi_{3K}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3) \left(2 \frac{u_0 - \alpha_1}{\alpha_3} - 1 \right) \end{aligned} \quad (8)$$

where M_1^2 and M_2^2 are the Borel parameters and

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2} \quad , \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} \quad .$$

We like to note that if we multiply the correlation function by the four-momentum p , we obtain

$$p^\mu T_\mu(p, q) = - \int d^4x e^{ip \cdot x} \left[\langle K^+(q) | T \left\{ \frac{\partial}{\partial x_\mu} j_\mu^K(x) j^{a_0}(0) \right\} | 0 \rangle + \delta(x_0) \langle K^+(q) | [j_\nu^K(x), j^{a_0}(0)] | 0 \rangle \right] , \quad (9)$$

where the second term results from the differentiation of the function $\theta(x_0)$ in the T-product of the currents. In the $SU(3)_{fl}$ limit $\partial^\mu j_\mu^K(x) = 0$, thus the first term on the right hand side of Eq. (9) vanishes. The second term can be calculated using the standard commutation relations, yielding for the correlation function the Ward identity

$$p^\mu T_\mu(p, q) = -i f_K q_\nu .$$

Similar Ward identities were considered in [11] where they were used to obtain relations between various pion distribution amplitudes.

Two-point correlation function satisfies a dispersion relation, therefore we can represent the invariant function as

$$T_1((p+q)^2, p^2) = \int \int ds ds' \frac{\rho^{had}(s, s')}{[s - (p+q)^2][s' - p^2]} \quad . \quad (10)$$

We saturate this dispersion relation by inserting a complete set of one hadron-states into the correlation function and we consider the single-particle K and a_0 states, this way we obtain

$$T_1((p+q)^2, p^2) = \frac{\langle 0 | j_\mu^K | K(p) \rangle \langle K K | a_0 \rangle \langle a_0(p+q) | j^{a_0} | 0 \rangle}{[(p+q)^2 - M_{a_0}^2](p^2 - M_K^2)} + \int_{s_0} ds \int_{s'_0} ds' \frac{\rho^{cont}(s, s')}{[s - (p+q)^2][s' - p^2]} \quad , \quad (11)$$

where the hadronic spectral density includes the contributions of higher resonances and the hadronic continuum. The matrix element $\langle K K | a_0 \rangle$ defines the coupling constant $g_{a_0 K^+ K^-}$

$$\langle K^+(q) K^-(p) | a_0(p+q) \rangle = g_{a_0 K^+ K^-} \quad (12)$$

and the current-particle matrix elements are given as

$$\langle a_0(p+q) | j^{a_0} | 0 \rangle = \lambda_{a_0} \quad , \quad (13)$$

$$\langle 0 | j_\mu^K | K(p) \rangle = i f_K p_\mu \quad . \quad (14)$$

After performing a similar double Borel transformation we obtain for the hadronic representation the result

$$T_1(M_1^2, M_2^2) = \lambda_{a_0} f_K g_{a_0 K^+ K^-} e^{-M_{a_0}^2/M_1^2} e^{-M_K^2/M_2^2} + \int_{s_0} ds \int_{s'_0} ds' \rho^{cont}(s, s') e^{-s/M_1^2} e^{-s'/M_2^2} . \quad (15)$$

The sum rule for the coupling constant $g_{a_0 K^+ K^-}$ then follows by equating the expressions $T_1(M_1^2, M_2^2)$ obtained for the invariant function $T_1((p+q)^2, p^2)$ by theoretical (Eq. 8) and by physical (Eq. 15) considerations. In order to do this we have to identify the second term in Eq. 15 representing the continuum contribution with a part of the term calculated theoretically in QCD, thus affecting the subtraction of the continuum. The prescription that has been suggested for this purpose [12,13] is based on the observation that the distribution amplitudes $\varphi_{pK}(u)$ and $\varphi_{\sigma K}(u)$ are polynomials in $(1-u)$, therefore we can write

$$-\varphi_p(u) + \frac{1}{6} \varphi'_\sigma(u) = \sum_{k=0}^N b_k (1-u)^k . \quad (16)$$

The continuum subtraction is affected in the leading twist 3 quark-antiquark term, since the contribution of the twist 3 quark-antiquark-gluon term in Eq. 8 is small, therefore finally we obtain the sum rule for the coupling constant $g_{a_0 K^+ K^-}$ in the form

$$g_{a_0 K^+ K^-} = \frac{1}{2\lambda_{a_0}} e^{M_{a_0}^2/M_1^2} e^{M_K^2/M_2^2} \left\{ \frac{M^2 M_K^2}{m_s} \sum_{k=0}^N b_k \left(\frac{M^2}{M_1^2} \right)^k \left[1 - e^{-A} \sum_{i=0}^k \frac{A^i}{i!} + e^{-A} \frac{M^2 M_K^2}{M_1^2 M_2^2} \frac{A^{(k+1)}}{(k+1)!} \right] \right. \\ \left. + \frac{f_{3K} M_K^2}{f_K} \int_0^{u_0} d\alpha_1 \int_{u_0-\alpha_1}^{1-\alpha_1} \frac{d\alpha_3}{\alpha_3} \varphi_{3K}(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3) \left(2 \frac{u_0-\alpha_1}{\alpha_3} - 1 \right) \right\} \quad (17)$$

where $A = s_0/M^2$ with s_0 the smallest continuum threshold.

In the numerical evaluation of the sum rule we use the twist 3 kaon distribution amplitudes given by [11]

$$\varphi_{pK}(u) = 1 + \left(30 \frac{f_{3K}}{\mu_K f_K} - \frac{5}{2} \frac{M_K^2}{\mu_K^2} \right) C_2^{1/2}(2u-1) \\ + \left[-3 \frac{f_{3K} \omega_{3K}}{\mu_K f_K} - \frac{27}{20} \frac{M_K^2}{\mu_K^2} (1 + 6a_2^K) \right] C_4^{3/2}(2u-1) \quad (18)$$

$$\varphi_{\sigma K}(u) = 6u\bar{u} \left\{ 1 + \left[5 \frac{f_{3K}}{\mu_K f_K} \left(1 - \frac{1}{10} \omega_{3K} \right) - \frac{7}{20} \frac{M_K^2}{\mu_K^2} \left(1 + \frac{12}{7} a_2^K \right) \right] C_2^{3/2}(2u-1) \right\} \quad (19)$$

$$\varphi_{3K}(u) = 360 \alpha_1 \alpha_2 \alpha_3^2 \left[1 + \frac{\omega_{3K}}{2} (7\alpha_3 - 3) \right] \quad (20)$$

where $C_m^k(2u-1)$ are the Gegenbauer polynomials. The overlap amplitude λ_{a_0} has been determined previously as $\lambda_{a_0} = (0.21 \pm 0.05) \text{ GeV}^2$ employing QCD sum rules method [8]. We also adopt the values at the renormalization scale 1 GeV $m_s(1\text{GeV}) = 150 \text{ MeV}$, in

$SU(3)_{fl}$ limit $f_{3K}(1GeV) = f_{3\pi}(1GeV) = 0.0035 \text{ GeV}^2$, $\omega_{3K}(1GeV) = -2.88$ and $f_K = 0.160 \text{ GeV}$ [11] with $M_K = 0.4937 \text{ MeV}$.

We then study the dependence of the sum rule for the coupling constant $g_{a_0 K^+ K^-}$ on the continuum threshold s_0 and on the Borel parameters M_1^2 and M_2^2 by considering independent variations of these parameters. We find that the sum rule is quite stable for the range of these parameters $1.00 \leq s_0 \leq 1.10 \text{ GeV}^2$, $0.7 \leq M_1^2 \leq 1.4 \text{ GeV}^2$ and $2 \leq M_2^2 \leq 6.0 \text{ GeV}^2$. By varying the values of the parameters s_0 , M_1^2 , and M_2^2 in these regions we obtain the result for the coupling constant $g_{a_0 K^+ K^-}$ as $4.4 \leq g_{a_0 K^+ K^-} \leq 5.6 \text{ GeV}$. The variation of the coupling constant as a function of the Borel parameters M_1^2 and M_2^2 , and the continuum threshold s_0 is shown in Fig. 1. We note that the sign that we obtain for the coupling constant $g_{a_0 K^+ K^-}$ is negative, that is $g_{a_0 K^+ K^-} < 0$.

There has been several previous estimations of the coupling constant $g_{a_0 K^+ K^-}$. The Novosibirsk SND collaboration data of the radiative decay $\phi \rightarrow \pi^0 \eta \gamma$ [14] was analyzed in a phenomenological framework in which the contributions of ρ meson, chiral loop and a_0 meson were considered, and the value $g_{a_0 K^+ K^-} = (-1.5 \pm 0.3) \text{ GeV}$ was obtained for this coupling constant [15]. The coupling constant thus obtained results in constructive interference between the contribution of different amplitudes. From the analysis of their experimental data of $\phi \rightarrow \pi^0 \eta \gamma$ decay, the KLOE collaboration estimated the coupling constant $g_{a_0 K^+ K^-}$ as $g_{a_0 K^+ K^-} = (2.3 \pm 0.7) \text{ GeV}$ [16]. A new analysis of the KLOE data on $\phi \rightarrow \pi^0 \eta \gamma$ decay, on the other hand, gives the result $g_{a_0 K^+ K^-} = (2.63^{+1.84}_{-1.28}) \text{ GeV}$ [17]. In this analysis the phase δ of the interference between $\phi \rightarrow a_0 \gamma \rightarrow \pi^0 \eta \gamma$ and $\phi \rightarrow \rho^0 \pi^0 \rightarrow \pi^0 \eta \gamma$ amplitudes was obtained as $\delta = 0$, which is in accordance with the constructive interference of the different amplitudes observed in the phenomenological analysis of $\phi \rightarrow \pi^0 \eta \gamma$ decay [14]. Our estimation of the coupling constant $g_{a_0 K^+ K^-}$ using light cone QCD sum rules results in a value somewhat larger than the previous determinations based on the analysis of $\phi \rightarrow \pi^0 \eta \gamma$ data. However, because of the intrinsic uncertainties of the light cone QCD sum rule method, our result can be taken only to indicate that the scalar a_0 meson state may have somewhat large strong coupling. Finally we note that the sign of the coupling constant $g_{f_0 K^+ K^-}$ was obtained as $g_{f_0 K^+ K^-} > 0$ in a previous light cone QCD sum rule determination of this coupling constant [13], and in a phenomenological analysis of $\phi \rightarrow \pi^0 \pi^0 \gamma$ decay [18]. Thus our result about $g_{a_0 K^+ K^-}$ that is $g_{a_0 K^+ K^-} < 0$, is consistent with the result obtained in the $q^2 \bar{q}^2$ model where $g_{f_0 K^+ K^-} = -g_{a_0 K^+ K^-}$ [2,5,19].

It has been argued that the ratio $R = BR(\phi \rightarrow f_0 \gamma) / BR(\phi \rightarrow a_0 \gamma)$ can provide insight into the structure of the scalar $a_0(980)$ and $f_0(980)$ mesons [4]. The KLOE collaboration obtained this ratio as [16]

$$R = \frac{BR(\phi \rightarrow f_0 \gamma)}{BR(\phi \rightarrow a_0 \gamma)} = 6.1 \pm 0.6 . \quad (21)$$

This ratio can be written, assuming the intermediate $K \bar{K}$ loop mechanism for these decays, in the form [4]

$$R = \frac{g_{f_0 K^+ K^-}^2}{g_{a_0 K^+ K^-}^2} \frac{F_{f_0}^2(R)}{F_{a_0}^2(R)} \cot^2 \theta \quad (22)$$

where the factors $F_{f_0}^2(R)$ and $F_{a_0}^2(R)$ are related to the spatial extensions of $f_0(980)$ and $a_0(980)$ mesons, and for point-like effective field theory calculations $F^2(R) = 1$. For a

spatially extended system with r.m.s. radius $R > O(\Lambda_{QCD}^{-1})$ the high momentum region of the integration is suppressed [4], resulting in the form factor with the property $F^2(R) < 1$, conversely $F^2(R) \rightarrow 1$ means a spatially compact system. The angle θ is the isospin mixing angle in the $f_0 - a_0$ system. If we use the result $6.2 \leq g_{f_0 K^+ K^-} \leq 7.8 \text{ GeV}$ obtained by a light cone QCD sum rule calculation [13], and our result $4.4 < |g_{f_0 K^2 K^-}| < 5.6 \text{ GeV}$ obtained by a similar light cone QCD sum rule method calculation we obtain

$$\cot\theta \frac{F_{f_0}(R)}{F_{a_0}(R)} \sim 1.8 \quad .$$

Since in light cone QCD sum rule calculations isospin is assumed to be exact, which corresponds to $\theta = 45^\circ$, we thus find $F_{f_0}(R)/F_{a_0}(R) \sim 1.8$ which seems to imply that the spatial extensions of $f_0(980)$ and $a_0(980)$ mesons are not equal. Therefore, also given the relation that we obtain about the relative sign between the coupling constants $g_{f_0 K^+ K^-}$ and $g_{a_0 K^+ K^-}$ in accordance with $q^2 \bar{q}^2$ model, we may suggest that our result supports the view that the structure of $a_0(980)$ and $f_0(980)$ mesons is a combination of a $K\bar{K}$ molecule with a compact $q^2 \bar{q}^2$ core with the spatial extension of $f_0(980)$ being more compact than that of $a_0(980)$ meson.

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FIGURES

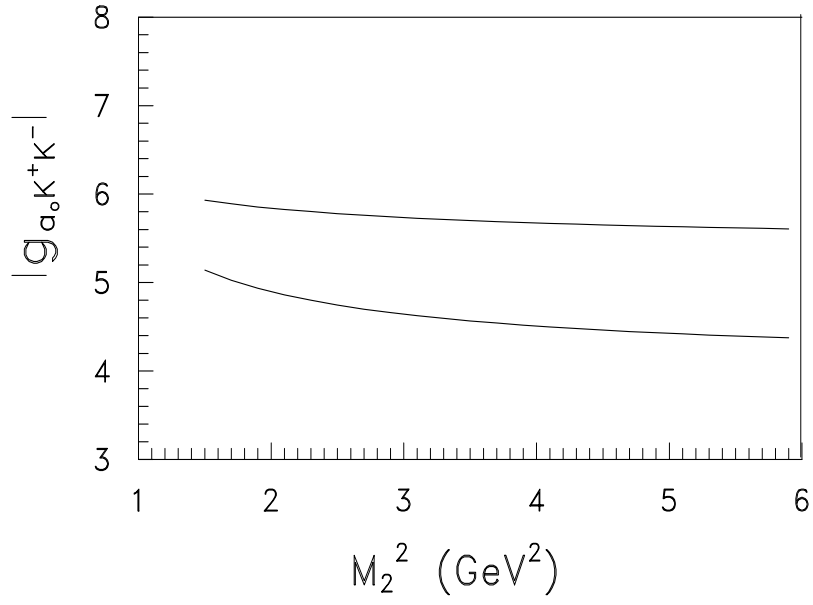


FIG. 1. The coupling constant $g_{a_0 K^+ K^-}$ as a function of the Borel parameter M_2^2 for different values of the threshold parameter s_0 and the Borel parameter M_1^2 . The curves denote the limits of the stability region.